

A DIRECT RECONSTRUCTION OF THE GAUGINO PARAMETERS WITH PHASES

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A simple algebraic algorithm is described to recover the (complex valued) gaugino/Higgsino basic parameters $\mu \equiv |\mu|e^{i\phi_\mu}$, $M_1 \equiv |M_1|e^{i\phi_{M_1}}$, $M_2 \equiv |M_2|$, directly in terms of (a minimal set of) neutralino and chargino masses, accurately measurable at future linear collider energies. An ambiguity in the M_1 reconstruction can be resolved by measuring the corresponding $e^+e^- \rightarrow \chi_1^0\chi_2^0$ cross-section. This approach should simplify the determination of allowed ranges of the gaugino parameters, in particular if only a partial set of gaugino masses were measured.

1 Introduction and Motivations

The unconstrained Minimal Supersymmetric extension of the Standard Model (MSSM)¹ involves a large number of arbitrary parameters, which furthermore may be in general complex valued, adding new sources of CP violation. Relatively large phases in the “flavour-blind” gaugino and/or Higgs sector are not excluded by present constraints^{2,3}, and may lead to drastic changes in the phenomenology of the Higgses and gauginos⁴⁻⁷, affecting the reconstruction from data of the structure of the SUSY and soft-SUSY breaking Lagrangian. We shall report here on a specific construction and algorithm^{5,6} to obtain the basic gaugino parameters in direct analytic form in terms of physical masses, including possible non-zero phases⁶. We illustrate in particular how this “dediagonalisation” approach can reveal in an easy way the non-trivial correlations among the chargino and neutralino sectors, exhibiting directly e.g. allowed domains for the neutralino masses, once some of the parameters of the chargino sector are determined⁴, or vice-versa.

2 Extracting μ , M_2 and M_1 from physical masses

Without loss of generality, in the unconstrained MSSM (i.e. no universality of the gaugino mass terms is assumed), we choose a phase convention such that the soft susy breaking $SU(2)_L$ Wino mass M_2 , is real. The chargino mass matrix thus reads

$$M_C = \begin{pmatrix} M_2 & \sqrt{2}m_W \sin\beta \\ \sqrt{2}m_W \cos\beta & |\mu|e^{i\Phi_\mu} \end{pmatrix} \quad (1)$$

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and the (squared) chargino masses are obtained as the eigenvalues of the $M_C^\dagger M_C$ matrix. The neutralino mass matrix with the relevant phases reads

$$M = \begin{pmatrix} |M_1|e^{i\Phi_{M_1}} & 0 & -m_Z s_W \cos \beta & m_Z s_W \sin \beta \\ 0 & M_2 & m_Z c_W \cos \beta & -m_Z c_W \sin \beta \\ -m_Z s_W \cos \beta & m_Z c_W \cos \beta & 0 & -|\mu|e^{i\Phi_\mu} \\ m_Z s_W \sin \beta & -m_Z c_W \sin \beta & -|\mu|e^{i\Phi_\mu} & 0 \end{pmatrix} \quad (2)$$

where $|M_1|e^{i\Phi_{M_1}}$ is the soft supersymmetry breaking $U(1)_Y$ Bino mass, while $|\mu|e^{i\Phi_\mu}$ parametrizes the supersymmetry conserving mixing of the two Higgs doublets.

2.1 Chargino de-diagonalization

Assuming as input $\tan \beta$, the two physical chargino masses and one mixing angle (say ϕ_L for definiteness), one can derive straightforwardly “inverted” expressions for $|\mu|$, Φ_μ and M_2 directly in terms of physical parameters^{6, b}:

$$|\mu|(M_2) = \left[\frac{1}{2} (\Sigma - 2m_W^2(1 + (-) \cos 2\beta - (+) \Delta \cos(2\phi_L)) \right]^{1/2} \quad (3)$$

$$\cos(\Phi_\mu) = 1 - \frac{M_{\chi_1}^2 M_{\chi_2}^2 - (M_2 |\mu| - m_W^2 \sin 2\beta)^2}{2m_W^2 M_2 |\mu| \sin 2\beta}. \quad (4)$$

where $\Sigma, \Delta \equiv M_{\chi_2}^2 \pm M_{\chi_1}^2$. Now, clearly not any input M_{χ_1} , M_{χ_2} will lead to $|\mu|$, $\cos \Phi_\mu$, M_2 values consistent with the obvious constraints $M_2, |\mu| \geq 0$, and $|\cos \Phi_\mu| \leq 1$. Depending on the actual values of M_{χ_1} , M_{χ_2} , this can give relatively strong constraints, as illustrated below.

2.2 Neutralino sector

We now determine $|M_1|$ and Φ_{M_1} for given M_2 , $|\mu|$, Φ_μ , $\tan \beta$ and two (arbitrary) input neutralino masses M_{N_1} , M_{N_2} ^c. Relatively simple explicit expressions for $|M_1|$ and Φ_{M_1} are obtained^{5,6} by considering the four invariants of the hermitian matrix $M^\dagger M$, given by the different coefficients of the characteristic polynomial $\det(M^\dagger M - M_N^2 I)$ in M_N^2 . However, the resulting system is quadratic in M_1 , which leads to an intrinsic twofold ambiguity in its determination^d. Also, quite similarly to the simpler chargino case discussed above, non-trivial constraints among the neutralino masses arise, simply because the system cannot always have a solution consistent with $|\cos \Phi_{M_1}|, |\sin \Phi_{M_1}| \leq 1$, for *any* input M_{N_1} , M_{N_2} .

2.3 Illustration of reconstruction from a complete set of data

In fig. 1a,b are plotted the output moduli and phases respectively, as functions of the chargino mixing angle ϕ_L , for typical input chargino and neutralino masses.

^bA method to extract the same parameters from chargino pair production is described in ref. ⁴.

^c For definiteness M_{N_1} , M_{N_2} are assumed to be the lightest and next to lightest. It should be clear, however, that any two masses among the four can be equivalently used as input.

^dAn alternative method to extract μ , M_1 , M_2 uniquely at linear collider energies was recently investigated in ref. ⁷, however using the full set of neutralino masses as data.

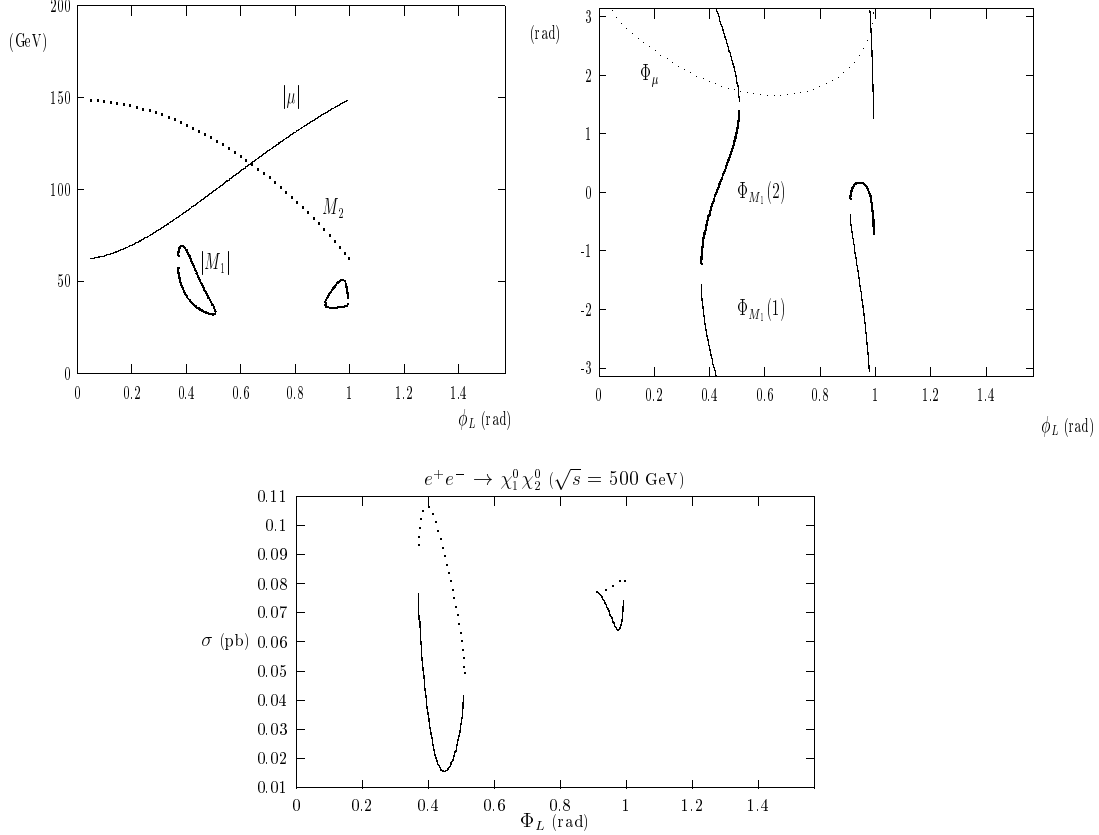


Figure 1: (a) (upper left) reconstruction of $|M_1|$, M_2 , $|\mu|$; (b) (upper right) reconstruction of Φ_μ , Φ_{M_1} . Physical input sample: $M_{\chi_1^+} = 80$ GeV, $M_{\chi_2^+} = 180$ GeV, $M_{N_1} = 40$, $M_{N_2} = 80$; $\tan \beta = 2$; (c) (bottom fig.) corresponding χ_1^0, χ_2^0 production cross-section for $\sqrt{s} = 500$ GeV, $m_{\tilde{e}_L, \tilde{e}_R} = 320, 300$ GeV.

The announced consistency correlations among the chargino and neutralino masses appear explicitly here in the fact that the ϕ_L domain where both sectors can consistently exist is quite narrow ($0.37 \lesssim \phi_L \lesssim 0.51$; $0.9 \lesssim \phi_L \lesssim 1$ (rad.)). Moreover, relatively moderate changes in the input mass values may easily result in narrower or even empty solution zones: changing e.g. M_{χ_1} only from 80 to 100 GeV, for the same values of the other input parameters, gives no consistent M_1 , M_2 , μ .

2.4 The $e^+e^- \rightarrow \chi_i^0 \chi_j^0$ cross-section with phases

Assuming non-zero M_1 , μ phases, it is crucial to analyze the changes implied in the relevant gaugino production cross-sections^{8–12,4,6}. Here we illustrate the neutralino 1,2 pair production at future linear e^+e^- collider, plotted in fig. 1c as a function of the chargino, neutralino masses and chargino mixing angle, for the same choice of parameters corresponding to fig. 1a,b and one choice of the selectron masses. One

first observes the generically rather important sensitivity of this cross-section to the phases⁹ (the plots in fig. 1c are functions of the mixing angle ϕ_L , which corresponds for fixed chargino and neutralino masses to varying Φ_μ and Φ_{M_1} , see corresponding fig. 1b). Moreover, and quite generically, the difference in the cross-section values for the two different M_1 solutions of fig.1 should easily resolve this two-fold ambiguity, provided the cross-section will be large enough to be measured with a reasonable accuracy. Furthermore, as a by-product of the inversion procedure, one can obtain direct correlations among e.g. the chargino masses and pair production, and the neutralino pair production cross-sections.

3 Constraints on basic parameter space from incomplete data

Let us consider now a less optimistic situation where only a partial knowledge of the physical input masses is assumed, and illustrate the kind of information that can be retrieved in this case. Accordingly, in fig. 2 we suppose that none of the neutralino masses are known, while the two chargino masses and $\tan\beta$ are fixed, and ϕ_L varies between $\phi_L \simeq 0.37$ and $\phi_L \simeq 0.51$ (rad), corresponding to the first consistent zone in fig. 1a,b. Consistency directly implies the pattern of correlation among the only possible physical neutralino masses, represented by the dotted regions (“butterflies”) in the figure. More precisely, a given set of the four neutralino masses is consistent only if any pair (M_{N_i}, M_{N_j}) of these masses corresponds to a point lying on one of the “butterflies”. From this requirement one can actually draw definite consequences such as the fact that each of the three allowed branches (along say the y-axis) can host only *one* pair (M_{N_i}, M_{N_j}) , etc. When $\tan\beta$ increases, the dotted butterflies are simply moving up or down along the diagonal $M_{N_i} = M_{N_j}$ line. We emphasize that within this algorithm, such correlations are very simply obtained from scanning over arbitrary values of the two input neutralino masses, checking the consistency relations only once for each M_{N_i} , M_{N_j} , (and eventually ϕ_L) input. Another example of possible application, that we do not illustrate here, is the direct determination of the allowed domains⁶ of the μ or M_1 phases, when a given range of the chargino and/or neutralino masses will be delineated by data.

4 Conclusions

A simple algebraic algorithm is derived to reconstruct the (unconstrained) gaugino sector Lagrangian parameters $\mu \equiv |\mu|e^{i\Phi_\mu}$, $M_2 \equiv |M_2|$ and $M_1 \equiv |M_1|e^{i\Phi_{M_1}}$, directly from the physical chargino and (some of the) neutralino masses. Our approach should be useful in particular in the case where only a subset of the minimal required input is available, where it exhibits in a more direct way the non trivial correlations among the physical chargino and neutralino parameters. These correlations exist even when the maximal possible phase freedom of the unconstrained MSSM parameter space is considered, and may be hidden or very tedious to extract in the more standard approach of systematic scanning over the basic parameters and fitting the data. Note finally that the inclusion of realistic mass measurement errors, given the accuracy level expected at a future linear collider^{12,7}, affects only very scarcely e.g. the shape of figs. 1, 2 and our reconstruction in general.

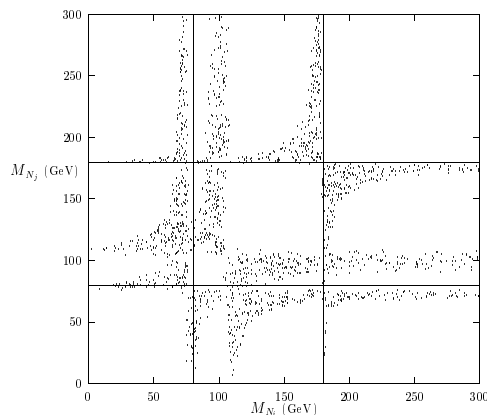


Figure 2: Correlations between arbitrary neutralino masses from consistency of the inversion. Input choice: $M_{\chi_1^+} = 80$ GeV, $M_{\chi_2^+} = 180$ GeV, $0.37 < \phi_L(\text{rad}) < 0.52$; $\tan \beta = 2$.

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